Lec 21: 10/31/2018

Thin Disk spectrum:

In order to calculate the disk spectrum, we first need to know whether it is optically thin or thick. The optical depth in the perpendicular direction is:

$$\tau_{\perp} = \int_{-\infty}^{+\infty} n_{(z)} \sigma_T \, dz = \frac{\sigma_T}{\mu m_H} \int_{-\infty}^{+\infty} \rho_{(z)} \, dz = \frac{\sigma_T}{\mu m_H} \Sigma$$

Using the values of σ_T and m_H (proton mass), and the expression for Σ in terms of the disk thickness (which we found in the previous lecture), we find that $\tau_{\perp} \sim 10^3$. As a result, the geometrically thin disk is optically thick, which implies that the emission must be blackbody. Therefore, the dissipation rate from the disk follows:

$$D_{(R)} = \sigma T^4_{(R)} \quad (\sigma: \text{Stefan Boltzmann constant})$$

After using the expression that we derived for $D_{(R)}$ last time, we find:

$$T_{(R)} = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma} \left[1 - \left(\frac{R_*}{R} \right)^{1/2} \right] \right\}^{\frac{1}{4}}$$

Over most of the disk $R \gg R_*$, and hence $T_{(R)} \propto R^{-3/4}$. The intensity of radiation at radius R is:

$$I_\nu(R) = \frac{\frac{2h\nu^3}{c^2}}{\exp\left(\frac{h\nu}{kT_{(R)}}\right) - 1}$$

An observer at a distance D away, with a line of sight making an angle i relative to the symmetry axis of the disk, will measure:

$$F_\nu = \int_{R_*}^{R_{out}} I_\nu \, d\Omega_{(R)}$$

Where:

$$d\Omega_{(R)} = \frac{2\pi R \, dR \cos i}{D^2}$$

Thus:

$$F_\nu = \frac{4\pi h\nu^3 \cos i}{c^2 D^2} \int_{R_*}^{R_{out}} \frac{R \, dR}{\exp\left[\frac{h\nu}{kT_{(R)}}\right] - 1}$$

As usual, it is useful to find the asymptotic behavior of F_ν. In the Rayleigh-Jeans limit, $h\nu << kT(R_{out})$, which implies that:

$$F_\nu^{RJ} \propto \nu^2$$

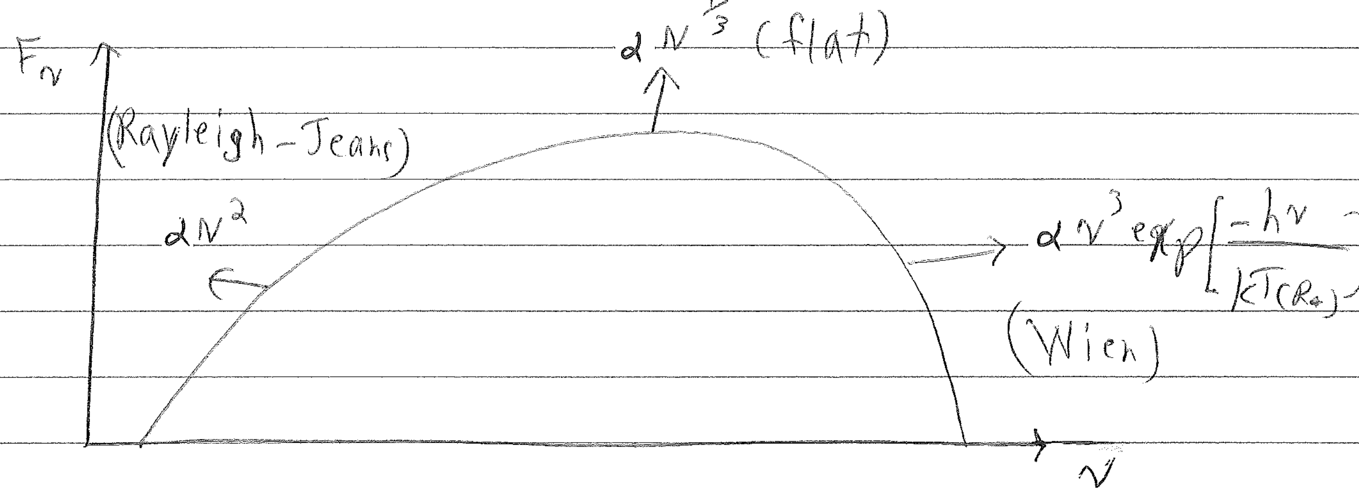
In the Wien limit, defined as $h\nu >> kT(R_*)$, we have:

$$F_\nu^{W} \propto \nu^3 \exp\left[\frac{-h\nu}{kT(R_*)}\right]$$

In this limit, the flux is dominated by the hottest portion of the disk near R_*. In between the two limits, we have:

$$F_\nu \propto \nu^{\frac{1}{3}} \int_0^\infty \frac{\eta^{\frac{5}{3}}}{e^\eta - 1} d\eta \propto \nu^{1/3} \quad \left(\eta \equiv \frac{h\nu}{kT(R)} \simeq \frac{h\nu}{kT(R_*)}\left(\frac{R}{R_*}\right)^{-3/4}\right)$$

In the Rayleigh-Jeans and Wien limits, the spectrum is that of a single blackbody, although at different temperatures, $T(R_{out})$ and $T(R_*)$ respectively. Between the two limits, the spectrum is the sum of blackbodies, which results in a segment $\propto \nu^{1/3}$ (so-called "flat" segment). The spectrum looks like as follows:



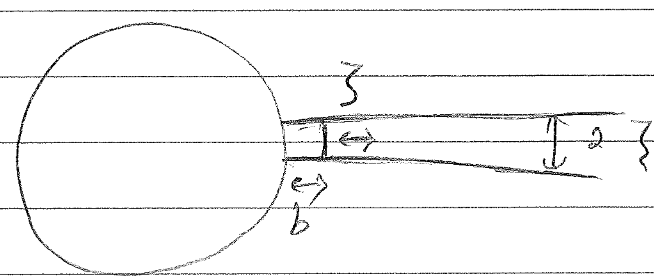
It can be distinguished from those of other thermal sources by its stretched out appearance. It is usually not difficult to identify a spectrum belonging to a thin disk once it has been assembled from multi-frequency observations. We note that the Wien region gives information about $T(R_*)$, while the Rayleigh-Jeans region contains information about $T(R_{out})$ → hence R_{out}, in the transition to the middle flat region.

Boundary Layers:

Let us now study the role played in high energy astrophysics by the transition region between the inner edge of the disk and the surface of the compact object in more detail.

Since $v_R \ll v_\phi$, the angular velocity $\Omega(R)$ in the disk remains very close to its Keplerian value until the matter reaches just outside the surface of the star at $R = R_*$. Within a boundary layer of radial extent b, Ω must decrease from $\Omega_K(R_*)$ to $\Omega(R_*) < \Omega_K(R_*)$. For a very slow rotation (as compared with the Keplerian velocity) at R_*, the boundary layer must be in hydrostatic equilibrium in the radial direction:

$$\frac{1}{\rho}\frac{\partial P}{\partial R} \simeq -\frac{GM}{R_*^2}$$



$$P \sim c_s^2 \rho$$

Thus:

$$\frac{c_s^2}{b} \sim \frac{GM}{R_*^2}$$

We also have hydrostatic equilibrium in the vertical direction (which was discussed last time):

$$\frac{1}{\rho}\frac{\partial P}{\partial z} \approx \frac{-GMz}{R_*^3} \Rightarrow \frac{P}{\rho \zeta} \sim \frac{GM\zeta}{R_*^3}$$

We therefore find:

$$\zeta \sim c_s R_* \left(\frac{R_*}{GM}\right)^{1/2} \Rightarrow b \sim \frac{\zeta^2}{R_*} = \left(\frac{\zeta}{R_*}\right)^2 R_* \ll R_*$$

The total luminosity produced by the disk is:

$$L_d = 2\int_{R_*}^{R_{out}} D(R)\, 2\pi R\, dR = \frac{3GM\dot{M}}{2}\int_{R_*}^{R_{out}} \left[1-\left(\frac{R_*}{R}\right)^{1/2}\right]\frac{dR}{R^2}$$

$$\Rightarrow L_d \approx \frac{GM\dot{M}}{2R_*}$$

On the other hand, the total gravitational energy released in the accretion is:

$$L_{acc} \approx \frac{GM\dot{M}}{R_*}$$

This implies that the power emitted from transition at the boundary layer is actually half of the total available power:

$$L_b = L_{acc} - L_d \approx \frac{GM\dot{M}}{2R_*}$$

The radiation emitted by the boundary layer is usually in the form of X-ray. It emerges through a region of radial extent $\sim \zeta$ on the two disk faces. This results in a blackbody radiation with temperature T_b, where:

$$4\pi R_* \zeta \sigma T_b^4 \approx \frac{GM\dot{M}}{2R_*}$$

This results in:

$$T_b \approx \left(\frac{GM\dot{M}}{8\pi R_*^2 \zeta \sigma}\right)^{\frac{1}{4}} \approx \left(\frac{R_*}{\zeta}\right)^{\frac{1}{4}} T_{(R_*)}$$

After using the relation for ζ, which we obtained in the previous lecture, we find:

$$T_b \approx \left[\frac{T_0}{T_{(R_*)}}\right]^{\frac{1}{8}} T_{(R_*)} \qquad \left(T_0 = \frac{GM\mu m_H}{kR_*}\right)$$

In a neutron star:

$M \simeq M_\odot$, $R_* \simeq 10\,km \Rightarrow T_0 \simeq 3\times10^{11}\,K$

Hence:

$T_b \simeq 3.6\alpha\, T(R_*) \simeq 3\times10^{7}\,K$

At this temperature, the boundary layer can produce a significant blackbody component above the regular disk spectrum. The combination of a stretched out thin disk spectrum plus a single (and hotter) blackbody component is indeed seen often in low mass X-ray binaries.